SHM Sensor Placement Optimization Under Uncertainty

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Abstract

Structural Health Monitoring (SHM) systems that report in real-time a flight vehicle's condition are central to meeting the demanding goals of increasing flight vehicle safety and reliability, while reducing costs. To detect damage with maximum probability, sensors must be placed optimally. This study develops a methodology for sensor placement optimization (SPO) for SHM systems under uncertainty. To achieve this, the structural component under consideration is analyzed via a finite element method (FEM) and model input uncertainties are included via random processes and fields. Probabilistic FEM analyses are performed to determine the model output variability and using their results, damage detection procedures such as feature extraction and state classification are applied to assess the current structural state of the component. Repeating these two steps using both healthy and damaged structural models allows for reliability estimates of a given sensor layout. Finally, SPO is achieved to maximize the reliability of damage detection. The sensor layout design of a thermal protection system component is presented to illustrate the proposed methodology.

1 Introduction

SHM systems that report in real-time the flight vehicle's condition in terms of stresses, displacements, extent of damage etc. are central to meeting the demanding goals of increasing flight vehicle safety and reliability, while simultaneously reducing vehicle-operating and maintenance costs [1]. The sensing components of the SHM system must be the most reliable components onboard the structure, as well as small, lightweight, and energy efficient, in order to be incorporated into the flight vehicle design with minimal impact on the structure's performance. The structural behavior of the flight vehicle is inherently random due to the uncertainties in the flight environment and system properties. A probabilistic structural analysis that includes the uncertainties associated with geometry, loads, and material properties is vital towards the success of optimal structural design.

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Such an analysis requires the combination of finite element analysis with uncertainty quantification methods. In addition, the sensors need to be optimally placed in order for the SHM system to detect with high probability any structural damage before it turns critical. While many advances have been made in terms of sensor technology, damage detection algorithms, structural reliability, and deterministic optimization methods, recent research on probabilistic modeling and design optimization under uncertainty also needs to be incorporated for effective sensor placement.

This paper develops a methodology to integrate the advances in the above individual disciplines to achieve optimum design under uncertainty of a SHM system sensor array. The proposed methodology includes the following steps: (1) structural modeling and analysis, (2) probabilistic analysis, (3) damage detection, and (4) sensor placement optimization. Section 2 describes the methodology, while Section 3 provides a numerical example.

2 General Methodology

2.1 Structural Modeling and Analysis

For most realistic structural systems, the response due to various loads cannot be determined via a closed-form expression of the input variables, but must be computed through numerical procedures such as finite element analysis (FEA). Other reasons to consider FEA may include (a) the reduction of prototype testing and (b) the simulation of designs that are not suitable for prototype testing [2]. Several commercially available finite element software packages are Patran/Nastran, Ansys, and Abaqus. Open source codes include CalculiX [3] and Impact [4]. Regardless of the software package used, the structural simulation must capture all physical phenomena and include all relevant input parameters. The appropriate analysis may include linear, nonlinear, and/or coupled structural-thermal simulations. Since FEA, which has a finite number of unknowns, can only approximate the response of the physical system, which has infinite unknowns, rigorous model verification and validation are required before practical implementation [2] (see Section 4).

2.2 Probabilistic and Reliability Analyses

Probabilistic finite element analysis incorporates uncertainty in model parameters. Due to the fact that most structural design parameters vary spatially or temporally and have random variability, model parameters such as distributed loads and material and geometric properties cannot be expressed as single random variables, but must be represented as random processes and random fields [5]. Random process and random field generation is a key step in this analysis. The Karhunen-Loeve Expansion (KLE) has been used extensively to simulate Gaussian random processes [6]. The wavelet transform method is an extension of the KLE simulation algorithm and is applicable to non-stationary Gaussian processes and fields [6]. Other random process and random field generation sequences include the Pierson-Moskowitz Wave Spectra, the JONSWAP spectra [7], Sakamoto's Polynomial Chaos Decomposition [8] and Shinozuka's Gaussian stochastic process formulation [9].

The two-dimensional Gaussian stochastic process in Fig. 1 was generated using the following spectral representation formulation, which is based partly on Shinozuka's [9] Gaussian stochastic process formulation and the Wiener-Khinchine relations [10]. The random field is simulated by the following series as N_1 and N_2 approach infinity.

$$g_o\left(x_1, x_2, \phi_{k_1,k_2}\right) = 2\sum_{k_1=1}^{N_1-1}\sum_{k_2=1}^{N_2-1}\left[\sqrt{2S\left(\omega_{k_1}\right)S\left(\omega_{k_2}\right)\Delta\omega_1\Delta\omega_2}\cos\left(\omega_{k_1}x_1 + \omega_{k_2}x_2 + \phi_{k_1,k_2}\right)\right]$$

, where $\Delta\omega_i = \omega_{u_i}/N_i$, $\omega_{k_i} = k_i\Delta\omega_i$, for $i = 1,2$. Here ω_{u_i} is the upper cutoff frequency beyond which $S\left(\omega_{k_i}\right)$ can be considered zero for all practical purposes. $S\left(\omega_{k_i}\right)$ is the two-sided power spectral density function of the random field in the i direction and ϕ_{k_1,k_2} an array containing the independent random phase angles uniformly distributed between 0 and 2π. N_i defines the number of terms to be included in the dual summation in the i direction. The random field in Fig. 1 utilizes the following power spectral density functions: $S\left(\omega_{k_i}\right) = \frac{1}{4}\sigma_i^2 b_i^3 \omega_{k_i}^2 \cdot \exp\left(-b_i\omega_{k_i}\right)$ for $i = 1,2$. Here σ_i is the standard deviation of the stochastic process in the i direction and b_i its "correlation distance."

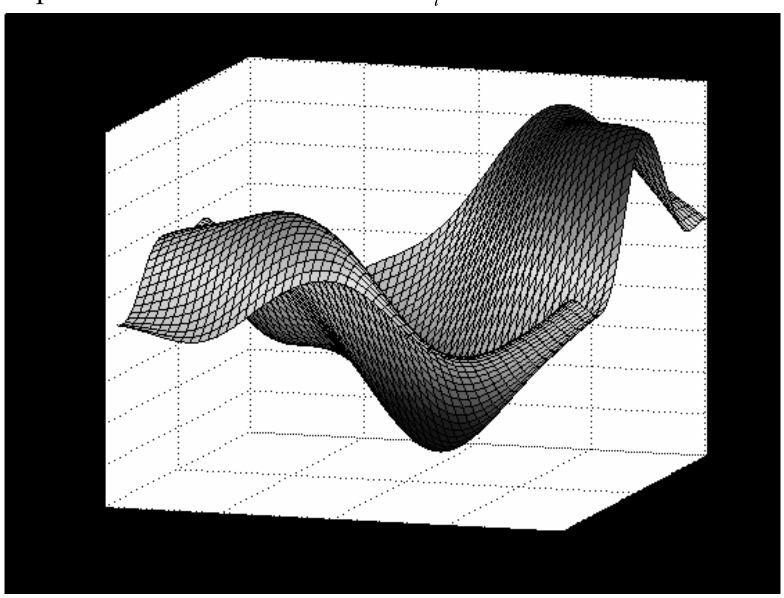

Figure 1 - Random Process Realization

Random field realizations, such as the one shown can be used to simulate component thickness, material moduli, and spatially distributed loads such as thermal and pressure loading. Linking random process/field realizations to spatially or temporally distributed model inputs allows the inclusion of their uncertainty in FEM analyses.

Once the stochastic parameters of the finite element model are randomly generated, multiple finite element analyses are used to generate statistical and/or sensitivity information with respect to the stochastic inputs, on output quantities such as stress, strain, or deformation at each possible sensor location i. For all practical purposes, each node of the finite element model represents a possible sensor location. Sensitivity information on other parameters such as mode shapes or stiffness deterioration may also be generated via FEM models that take material degradation into account and are analyzed with respect to time or load cycles.

Additional analysis is needed to estimate the probability of correctly identifying the structural state of a component for a given sensor layout x $\left(i.e.\ P_x = P(\text{correct structural classification} \mid \text{sensor layout } x)\right)$. This can be accomplished via any appropriate diagnostics signal analysis procedure. The signal analysis procedure employed in this study, utilizes the feature extraction and state classification methodologies defined in [8]. Repeated analyses using different realizations of the random inputs to healthy and damaged structural FEM models and their respective state classification constructs a classification matrix from which the probability of detecting damage as well as the probability of false alarm can be estimated. Further details of such a procedure are given in Section 2.3.

2.3 Damage Detection Algorithms

Damage detection and location identification algorithms include wavelet-based approaches [11], two-stage modal frequency analysis [12], and methods for eddy-current-based damage detection [13]. Also, several methods for structural damage detection use energy dissipative devices, which guarantee closed-loop stability [14]. Property matrix updating, nonlinear response analysis, and damage detection using neural networks are all methods used to manipulate the information gathered by SHM systems for decision making. However, most structural damage detection methods and algorithms found in the literature examine the changes in the measured structural vibration response and analyze modal frequencies, mode shapes, and/or flexibility coefficients of the monitored structure [15]. This can be achieved either actively or passively, where active damage detection algorithms utilize the system response to an auxiliary excitation and passive methodologies utilize solely the response to operational vibrations. A comprehensive review of damage detection and location identification algorithms is provided in [15].

2.4 Sensor Placement Optimization

Studies by Padula [16, 17, 18] and Raich [19] have examined the problems and issues involved with SPO. Integer and combinatorial optimization methods have been used to optimize the placement of actuators for vibration control and noise attenuation. In addition genetic algorithms for the optimization of sensor layouts [19] have been proposed. The multivariate simultaneous perturbation stochastic approximation (SPSA) method allows for the inclusion of noise in function evaluations or experimental measurements and shows to be very efficient for large-dimensional problems [20]. In general, however, the methods used for SPO can be stated as "given a set of n possible locations, find the subset of a locations, where $a << n$, which provide the best possible performance" [16].

Another approach to optimizing sensor placement is implemented in the software package Snobfit (Stable Noisy Optimization by Branch and Fit) [21]. This optimization scheme is designed for bound-constrained optimization of noisy objective functions, which are costly to evaluate due to computational or experimental complexity. The major advantage of the Snobfit algorithm is that it does not require a previously determined set of candidate sensor locations, but rather considers the following optimization problem.

$$\begin{aligned} &\min f(x) \\ &s.t.\ \ x \in [u,v] \end{aligned} \qquad (1)$$

where x is continuous and $[u,v]$ is a bounded box in $\Re^n$ with a nonempty interior [19].

The underlying idea of the optimization formulation is to identify a sensor layout x that will maximize the probability of correctly classifying the structure as either healthy or damaged (i.e. classify the structure as healthy when it is indeed healthy and as damaged when it is damaged). Here x represents a vector containing the coordinates of the SHM sensors for a given layout. From the probabilistic analysis described above and a diagnostics signal analysis procedure, the probability of correctly identifying the structural state P_x is known. This allows the optimization formulation given in Equation 1 to be utilized, where $f(x) = -P_x$ and $[u,v]$ are the geometric constraints on x given by the physical dimensions of the structure. This formulation is solved using Snobfit to determine the optimum sensor layout (see section 3). Other applicable optimization algorithms may be substituted for Snobfit.

3 Application of Methodology

For purpose of illustration, the above methodology was implemented using the following example problem. The structure under consideration is the thermal protection system (TPS) component that is described in detail in [22] and shown in Fig. 2

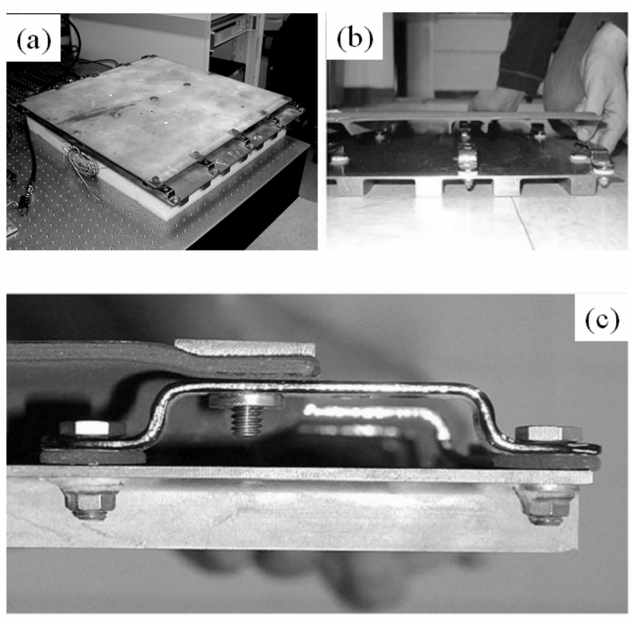

Figure 2 - TPS component; (a) general setup; (b) side view showing C-C plate, ribs, brackets, and backing structure; (c) bracket detail [8].

3.1 Structural Modeling and Analysis

The structure under consideration is modeled using the commercial finite element software Ansys (Release 7.0 – Licensed to Vanderbilt University). A portion of the FEM model is shown in Fig. 3. The model consists of a carbon-carbon (C-C) heat resistant panel that is attached to a titanium backing structure via 15 silicon-carbide-carbon (SiC-C) brackets and 45 bolts. An experimental setup of this TPS component exists at the Wright Patterson Air Force Base (WPAFB) Air Force Research Laboratory (AFRL) and is currently undergoing testing [8]. The FEM model consists of approximately 15,000 Shell-43 (carbon-carbon panel and brackets) and Shell-63 (titanium backing structure) 4-noded shell elements. The analysis is transient and includes a dynamic mechanical load consisting of a sinusoidal swept frequency deformation excitation, exciting the structure from 0 to 3000Hz in 1.0 seconds. This excitation represents the auxiliary input used with active damage detection algorithms. Due to the high frequency of the excitation function, the time step of the transient analysis approaches zero, making the analysis quite time-consuming. Higher accuracy requires higher fidelity, which increases computational complexity and makes the model computationally expensive to run. Each finite element analysis requires approximately 24 to 36 hours of processing time with dual 2.0 GHz Xeon or Opteron processors and produces a results file of about 4 GB in size. Parallel computing runs on multiple processors, at Vanderbilt's Advanced Computing Center for Research and Education (ACCRE) – a high performance computing facility – are utilized for Monte Carlo simulations and probabilistic finite element analysis.

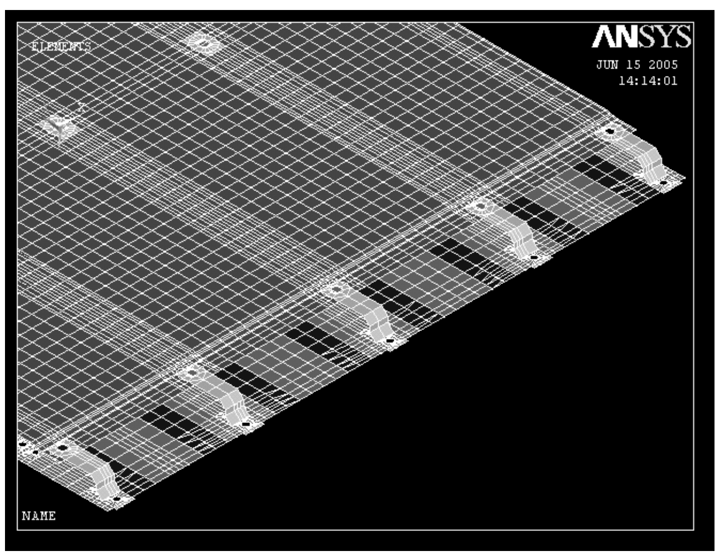

Figure 3 - Finite element model of TPS component.

3.2 Probabilistic Analysis

In the current example the following FEM model inputs are considered stochastic: 1) thickness of C-C TPS plate, SiC-C brackets, and titanium backing structure, 2) material properties including Young's Modulus, Shear Moduli, Poison's Ratio, and density, 3) amplitude of the excitation function, and 4) applied temperature field. The thickness parameters and material properties are modeled as

Gaussian random fields using the formulation presented in Section 2.2. The amplitudes of the excitation function and the applied temperature field are modeled as normal random variables. The coefficients of variations (COV) of all random fields are assumed to equal 0.05. The COV of each of the normal random variables is taken as 0.25.

Repeatedly performing deterministic finite element analyses using realizations of the stochastic model inputs provides data for statistical analysis. For the example at hand, 200 simulations using 200 random realizations of the inputs were processed. 100 of these realizations were used to simulate "healthy" (i.e. no structural damage) runs. The remaining 100 realizations are inputs to "damaged" (i.e. fastener damage at one single bracket) runs. These two sets of simulations were used for damage detection and state classification as described in Section 3.3.

3.3 Damage Detection and State Classification

Figure 4 shows the top view of the TPS component with a typical sensor layout, where sensor location S-1 is the point of input excitation, which remains stationary, and sensor locations S-2, S-3, and S-4 are the points of sensing and are variable. Also shown in Fig. 4 are the locations of the 15 brackets connecting the C-C panel to the titanium backing structure, and the location of the fastener damage at bracket number 11, which simulates the "damaged" structural state.

From the pool of simulation output consisting of temporal displacement data, a set of features is extracted. Features are characteristics unique to a signal generated under a given set of parameters. The set of features utilized for this example problem is composed of both time and frequency domain features. Time domain features are derived from the auto- and cross-correlations of the sensor responses collected at three different locations. Utilizing temporal data from three sensors, three auto-correlation and three cross-correlation functions may be computed. The shapes of the envelopes of these functions are determined and their second through fourth central moments are computed. This yields 18 temporal features. The frequency domain features are obtained from the first through fourth central moments of the modified coherence function defined in [22]. This yields additional 12 features providing a 30-dimensional feature vector y .

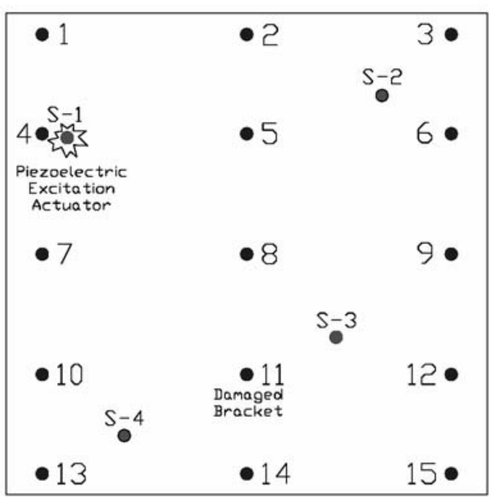

Figure 4 – TPS Plate with Typical Sensor Layout, Actuator, and Fastener Damage Locations

The above defined feature vector y is then used for state classification. The state classifier utilized in this work is derived from Bayes decision theory and minimizes the probability of classification error [23]. The discriminant function for each structural state ("healthy" and "damaged") is the Mahalanobis distance as given in Equation 2.

$$d_j(y) = (y - \mu_j)^t \Sigma_j^{-1} (y - \mu_j) \qquad (2)$$

where j indexes the structural state, y is the feature vector stemming from the simulation currently to be classified, and μ_j and Σ_j are the mean vector and covariance matrix of the entire data set of a given structural state minus the simulation currently being classified. Since the Mahalanobis distance requires the determination of the inverse of Σ_j it is necessary that the feature covariance matrix be non-singular. To assure this for the problem at hand, several features had to be discarded from the feature set, leaving a 12-dimensional feature vector for state classification.

State classification is continued by evaluating each discriminant function for each of the 100 "healthy" and 100 "damaged" simulations and assigning the state according to the discriminant function with the smallest value. This yields a classification matrix corresponding to a given sensor layout. Table 1 below shows the classification matrix for sensor array 23.

Sensor Array #23	Classified as "Healthy"	Classified as "Damaged"
100 "Healthy" Simulations	76	24
100 "Damaged" Simulations	9	91

Table 1 – Typical Classification Matrix

From this classification matrix, the probability of correct damage detection as well as the probability of false alarm can be calculated for a given sensor array. For example, sensor array #23 yields a probability of correct damage detection $P_{23}(\text{Correct Damage Detection}) = 91/100 = 0.91 = 91\%$ and a probability of false alarm $P_{23}(\text{False Alarm}) = 24/100 = 0.24 = 24\%$. Repeating the above for different sensor arrays, yields varying probabilities of detection, which allows for optimization (maximizing the probability of correct damage detection) of the SHM system with respect to the position of sensors.

3.4 Sensor Placement Optimization

The software package Snobfit [21], as programmed in Matlab, is used to iteratively solve the optimization formulation given by Equation 1. The Snobfit algorithm partitions the search domain bounded by the box $[u, v]$ into a set of sub boxes such that each contains exactly one point. A novel branching algorithm is utilized. Snobfit then builds local quadratic models, around the current best point, x^{best}, and around all other points. Two different types of quadratic models are used: a Hessian fit around the best point, x^{best}, and around all other points a quadratic fit using as Hessian a suitable multiple of the Hessian matrix used in the model around x^{best}. The algorithm then suggests a user-specified number of evaluation points to be used in the next iteration of the optimization. With the aid of trust regions and the model around x^{best}, two such points are generated. With the aid of the local quadratic models around all other points whose function values are predicted to be good, several further evaluations points are suggested. In addition points in unexplored regions of the search domain and points randomly chosen such that the distance from already sampled points is maximum are provided [21].

The function can be evaluated at these suggested points and other locations for further Snobfit iterations. Due to the fact that the structural FEM model described in section 3.1 has finite fidelity and temporal data is only available at the nodes, the locations Snobfit requests are substituted with the nearest neighboring nodal locations. The damage detection and state classification procedure described in Section 3.3 is performed for these requested sensor layouts to determine their corresponding P_x. Snobfit then continues optimizing Equation 1 with vectors u and v confining sensors S-2, S-3, and S-4 to their respective quadrants of the TPS component. Iterations continue until a convergence criterion is reached, such as when the sum of effective displacements of sensors S-2, S-3, and S-4 between consecutive iterations has reached some sufficiently small value.

4 Conclusion and Future Work

A methodology for sensor placement optimization under uncertainty is outlined in this paper. The method consists of four components: (1) structural modeling and analysis, (2) probabilistic analysis, (3) damage detection, and (4) sensor placement optimization. The methodology is applied to the optimization of a SHM system for a TPS component. Future work consists of model verification and validation, which are the primary means of assessing the accuracy of models and computer simulations and build confidence and credibility in simulation-based research [24, 25]. The experimental setup at AFRL-WPAFB of the TPS component is to be used to validate the FEM model. Future research also needs to incorporate sensor reliability and redundancy within the optimization methodology.

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