Title: Sensor Placement Design for SHM Under Uncertainty

Authors: R. F. Guratzsch
S. Mahadevan

ABSTRACT

Structural Health Monitoring (SHM) systems increase aerospace vehicle safety and reliability, while reducing vehicle operating and maintenance costs. Many advances have been made in terms of sensors, materials, damage detection algorithms, structural reliability, finite element analysis, and deterministic sensor placement optimization (SPO) algorithms. Additional research needs to be focused on probabilistic modeling, analysis, and design, as well as SPO under uncertainty. This paper discusses a methodology to integrate the advances in various disciplines for the optimum design of SHM systems under uncertainty. This includes finite element analysis under varying loads and incorporation of uncertainty quantification methods. Validating the finite element model with experimental data and accounting for uncertainties of experimental measurements and model predictions are also part of this analysis. The SHM sensors need to be placed optimally in order to detect with high probability and reliability any structural damage before it turns critical. The proposed methodology achieves this objective by combining stochastic finite element analysis, structural damage detection analysis, and SPO.

1 INTRODUCTION

SHM systems that report in real-time the flight vehicle's condition in terms of reactions, stresses, and displacements, are central to meeting the demanding goals of increasing flight vehicle safety and reliability, while reducing vehicle operating and maintenance costs [1]. The SHM system must be the most reliable sub-system on board the flight structure in order to make incorporation into existing flight vehicle designs possible. The structural behavior of next generation flight vehicles is inherently random due to the uncertainties in the flight environment and a probabilistic structural analysis that includes the uncertainties associated with geometry, loads, and material properties is vital toward the success of the design.

Robert F. Guratzsch, Vanderbilt University, Graduate Student, Department of Civil and Environmental Engineering, Vanderbilt University, VU Station B 351831, Nashville, TN 37235; Phone: 812.887.1132; E-mail: Robert.F.Guratzsch@Vanderbilt.edu.

Sankaran Mahadevan, Vanderbilt University, Professor of Civil and Environmental Engineering, Vanderbilt University, VU Station B 351831, Nashville, TN 37235; Phone: 615.322.3040; Fax: 615.322.3365; E-mail: Sankaran.Mahadevan@Vanderbilt.edu.

This includes the development of a finite element model, incorporating uncertainty quantification methods, and performing the necessary post-processing analysis. In addition, the sensors need to be optimally placed in order for the SHM system to detect with high probability any structural damage before it turns perilous. While many advances have been made in terms of sensor technology, damage detection algorithms, structural reliability, and deterministic SPO schemes, much additional research needs to be focused on probabilistic modeling, probabilistic analysis and design, as well as on SPO under uncertainty.

This paper defines a methodology for integrating the advances in various individual disciplines for optimum sensor layout design of structural health monitoring systems under uncertainty. The methodology aims at maximizing the probability of detecting damage with respect to the location of SHM sensors. This includes the following steps: (1) structural simulation and model validation, (2) probabilistic analysis, (3) damage detection, and (4) sensor placement optimization. Section 2 of this paper defines the general methodology, while section 3 provides a numerical example to demonstrate optimization under uncertainty.

2 GENERAL METHODOLOGY

2.1 Structural Simulation and Model Validation

For most realistic structures, the response due to various loads cannot be determined via a closed-form function of the input variables. The response of the structure under consideration must be computed through numerical procedures such as a finite element method (FEM). Several commercially available finite element software packages are Patran/Nastran/Dytran [2,3], Ansys [4], and Abaqus [5]. Open source codes, which can also be utilized, include CalculiX [6] and Impact [7]. Regardless of the software package used, the structural simulation must capture all physical phenomena and include all relevant input parameters. The appropriate analysis may include linear, nonlinear, and/or coupled structural-thermal simulations.

In addition, model verification and validation is of extreme importance and is the primary mean of assessing the accuracy of models and computer simulations. Validation metrics that can be utilized to build confidence and credibility in models are the modal assurance criterion (MAC) along with the modal scale factor (MSF) [8] and the model reliability metric (MRM) [9]. Validation of numerical models by comparison against experimental observations has to account for errors and uncertainties in both model prediction and measured observation. MRM includes the probabilistic nature of and the uncertainty associated with model predictions and laboratory observations (i.e. standard deviations, distributions, etc.).

2.2 Probabilistic Analyses

Probabilistic FEM analysis of analytical models incorporates uncertainty via the substitution of discretized random fields and processes as model parameters. Due to the fact that most structural design parameters vary spatially, model parameters such as distributed loads and material and geometric properties cannot be expressed as single random variables, but must be represented as random processes and random fields [10]. Random process/field generation is a key step in this component of the methodology and several generation sequences are available.

Once the finite element model parameters are generated via the discretization of random fields and processes and linked to the FEM model, a probabilistic finite element analysis is used to generate statistical and/or sensitivity information on stress, strain, or deformation at each possible sensor location i. For practical purposes, each node of the finite element model can represent a possible sensor location.

2.3 Damage Detection Algorithms

Damage detection and location identification algorithms are numerous and include several different approaches [11,12]. However, most structural damage detection methods and algorithms found in the literature examine the changes in the measured structural vibration response and analyze the modal frequencies, mode shapes, and flexibility coefficients of the structure [13]. A comprehensive review of the state of the art damage detection and location identification algorithms is provided in [13].

From a probabilistic FEM analysis the stochastic nature of stresses, strains, and deformations is known for all possible sensor locations. Additional analysis is needed to estimate the probability of correctly identifying the structural state of a component for a given sensor layout, x (i.e. $P_x = P(\text{correct structural classification} \mid \text{sensor layout } x)$). This is accomplished via any appropriate diagnostics signal analysis procedure (i.e. damage detection algorithm). The signal analysis procedure employed in this study, utilizes the feature extraction and state classification methodologies defined in [14]. Repeated analyses using different realizations of the random inputs to healthy and damaged FEM models and their respective state classification constructs a confusion matrix from which the probability of detecting damage and the probability of false alarm is estimated. Details of such a procedure are given in Section 3.3.

2.4 Sensor Placement Optimization

Recent studies by Padula [15,16,17] and Raich [18] have examined the problems and issues involved with SPO. In general, the methods used can be stated as "given a set of *n* candidate locations, find *a* locations, where *a* << *n*, which provide the best possible performance" [15]. A common critique of this line of attack is the requirement to specify *n* candidate sensor locations.

An alternate approach to optimizing sensor placement is implemented in Snobfit (Stable Noisy Optimization by Branch and Fit) [19], an optimization scheme that is designed for bound-constrained optimization of noisy objective functions, which are costly to evaluate. Snobfit does not require a priory set of candidate sensor locations, but rather considers the following optimization problem.

$$\begin{aligned} &\min f(x) \\ &s.t.\ \ x \in [u,v] \end{aligned} \tag{1}$$

where x is continuous and $[u,v]$ is a bounded box in $\Re^n$ with a nonempty interior [19]. Here x represents a vector containing the coordinates of the SHM sensors for a given layout. From the reliability analysis described above and a diagnostics signal analysis procedure, the probability of correctly identifying the structural state, P_x, is known. This allows the optimization formulation given in Equation (1) to be utilized, where $f(x) = -P_x$ and [*u*,*v*] are the geometric constraints on *x* given by the physical dimensions of the structure.

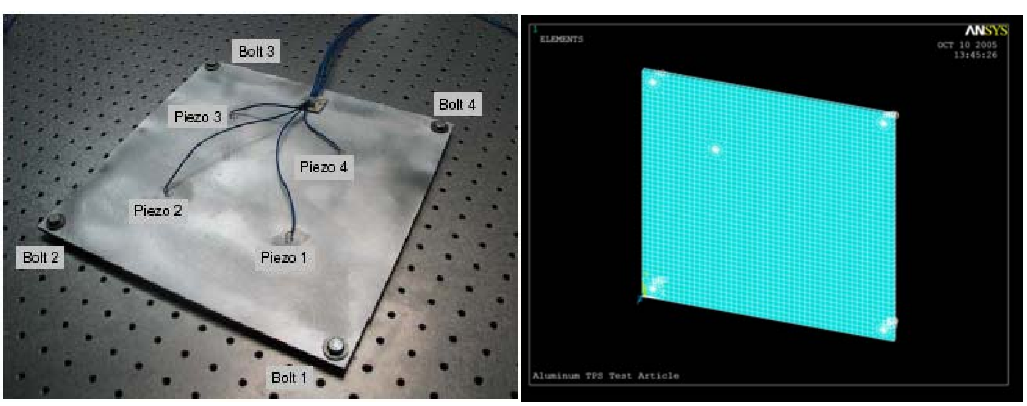
3 APPLICATION OF METHODOLOGY

For the purpose of illustration, the above methodology was implemented using the following example problem. The structure under consideration is a simplified thermal protection system (TPS) component that is described in detail in [20], and shown in Figure 2a. The test article consists of a heat-resistant, 0.25 inch thick aluminum plate, held in place via four 0.25 inch diameter bolts located 0.50 inches from the edges of the plate.

3.1 Structural Simulation and Model Validation

The structure under consideration is modeled using the commercial finite element software Ansys [4]. A portion of the FEM model is shown in Figure 2b. Four-noded shell elements and two-noded spring elements are utilized to model the aluminum plate and bolted boundary conditions. Approximately 3,300 nodes and 2,800 elements comprise the 19,836 degree of freedom (DOF) models. In Figure 2b, the four points located near the corners of the plate simulate the bolted boundary conditions, while the point near the center of the upper left quadrant of the plate simulates the piezoelectric actuator.

The analysis is transient and includes a dynamic mechanical load consisting of a sinusoidal frequency sweep, exciting the structure from 0 to 1500 Hz in approximately 2.0 seconds. This excitation represents the auxiliary input used with active damage detection algorithms. Due to the high frequency of the excitation function, a mode superposition transient (MSP) analysis was used to evaluate the FEM model.



a) b)

Figure 2 – a) Experimental Setup of TPS Test Article Showing Bolts and Piezoelectric Transducer Placement [20], b) Finite Element Model of TPS Component.

This study employs two metrics for validation purposes: the modal assurance criterion (MAC) [8] and the model reliability metric (MRM) [9]. These metrics are used to compare model predictions and experimental observations. For the problem at hand, MAC and MRM are used to compare both modal frequencies and mode shape vectors. Comparisons were performed for models of healthy and damaged test articles. These comparisons and their results are published in [21], and conclude that all model results were found to be highly correlated to experimental observations in regards to natural frequencies as well as mode shapes, such that the models can be considered validated with high reliability.

3.2 Probabilistic Analysis

In the current example, plate thickness, Young's modulus, Poison's ratio, and density are modeled as Gaussian random fields with independent, but equal correlation structures along orthogonal axes. A two-dimensional stochastic process was generated for these model inputs using the spectral representation as defined in Equation (4) via [22] and the Wiener-Khinchine relations [23]. The Gaussian random field $g_o(x_1, x_2, \phi)$ can be simulated by the following series as N_1 and N_2 approach infinity.

$$g_o(x_1, x_2, \phi) = 2\sum_{k_1=1}^{N_1-1}\sum_{k_2=1}^{N_2-1}\left[\sqrt{S(\omega_{k_1})S(\omega_{k_2})\Delta\omega_1\Delta\omega_2}\cos\left(\omega_{k_1}x_1 + \omega_{k_2}x_2 + \phi_{k_1,k_2}\right)\right] \qquad (4)$$

where $\Delta\omega_i = \omega_{u_i}/N_i$, $\omega_{k_i} = k_i\Delta\omega_i$, for $i = 1,2$. Here ω_{u_i} is the upper cutoff frequency beyond which $S(\omega_{k_i})$ is considered zero. $S(\omega_{k_i})$ is the two-sided power spectral density function of the random field in the i direction and ϕ_{k_1,k_2} an array containing the independent random phase angles uniformly distributed between 0 and 2π. N_i defines the number of terms to be included in the dual summation in the i direction. The random fields in this study utilize the following power spectral density functions: $S(\omega_{k_i}) = \frac{1}{4}\sigma_i^2 b_i^3 \omega_{k_i}^2 \cdot \exp(-b_i\omega_{k_i})$ for $i = 1,2$. Here σ_i is the standard deviation of the stochastic process in the i direction and b_i its corresponding "correlation distance."

For the random fields considered as FEM inputs to models of the test articles, $b_1 = b_2 = 3$ and $\sigma_1 = \sigma_2 = 1$, where the magnitude of $g_o(x_1, x_2, \phi)$ is scaled after the fact to match the mean and coefficient of variation (COV) of the random field to be simulated. $\omega_{u_1} = \omega_{u_2} = 5\pi$, while $N_1 = N_2 = 35$. Table I lists the means and COV used for each of the random fields simulated with Equation (4).

Table I - Mean and COV Values Used for Random Field Simulation

	Panel Thickness (in)	Young's Modulus (psi)	Poison's Ratio	Density (lb-mass/in^3)
Mean	0.2458	9.75E+06	0.3	2.59E-04
COV	0.05	0.05	0.05	0.05

Temperature uncertainty was included as a random variable uniformly distributed between 65 and 75 degrees Fahrenheit. The following temperature effect model was constructed via a quadratic regression analysis of data published by [24]:

$$F(t) = \left(-1.151525 \times 10^{-6}\right) \times t^2 + \left(2.75775 \times 10^{-5}\right) \times t + 1.00067 \qquad (5)$$

where $F(t)$ is a scale factor for Young's modulus and t is the plate temperature in degrees Fahrenheit.

Repeatedly evaluating deterministic finite element analyses using realizations of the model inputs provides a source of data for statistical analysis purposes. For the example at hand, 500 simulations using 500 realizations of the random inputs were executed; 100 simulations of the healthy model, 100 simulations of the model damaged at bolt 1, 100 simulations of the model damaged at bold 2, and so on. These 5 sets of simulations and their corresponding probabilistic characteristic are used for damage detection.

3.3 Damage Detection and State Classification

Figure 4 shows a typical sensor layout, where sensor location 1 is the point of input excitation and stationary, while sensor locations 2, 3, and 4 are the points of sensing and variable. Also shown in Figure 3 are the locations of the 4 bolts which hold the test structure in place and are the locations of fastener damage.

From the pool of simulation output of the probabilistic FEM analysis consisting of temporal displacement data, a set of features consisting of 300 measurements of the discrete Fourier transforms (DFTs) of the calculated Von Mises stress of the response signals of piezoelectric sensors 2, 3, and 4 (100 magnitudes each) are utilized. Dimensionality reduction is achieved via principal component analysis [25], where the original 300 measurements are projected into a lower dimensional space.

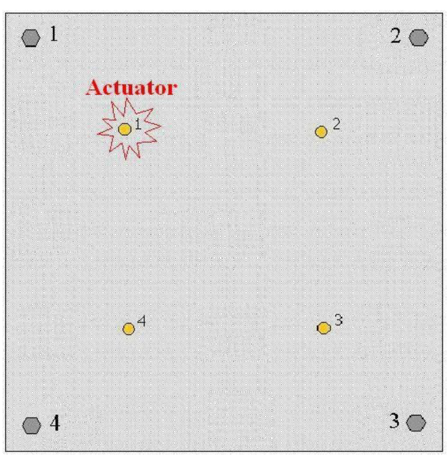


Figure 3 – TPS Test Plate with Typical Sensor Layout, Actuator, and Fastener Damage Locations

The above defined features are then used for state classification. The state classifier utilized in this work is derived from Bayes decision theory and minimizes the probability of classification error [26]. The discriminant functions, one for each structural state, are the Mahalanobis distance as given in Equation 6.

$$d_j(x) = (x - \mu_j)^t \Sigma_j^{-1} (x - \mu_j) \quad (6)$$

where j indexes the structural state, x is the feature vector to be classified, and μ_j and Σ_j are the mean vector and covariance matrix of the training data set of a given structural state. The training data consists of the responses of the first 50 simulations of each structural state.

State classification is continued by evaluating each discriminant function for each simulation of the testing data set and assigning the state according to the discriminant function with the smallest value. The testing data consists of the second 50 simulations of each structural state. This yields a classification matrix corresponding to a given sensor layout, from which the probability of correct damage detection as well as the probability of false alarm can be estimated. Repeating the above calculations for different sensor layouts allows for optimization (maximizing the probability of correct damage detection) of the SHM system with respect to the position of sensors. Training and testing data sets are reversed to achieve higher fidelity within the classification matrices.

The software package Snobfit [19], as programmed in Matlab, is used to solve the optimization formulation given by Equation 1 iteratively, based on domain partitioning and local quadratic models.

4 CONCLUSION

A methodology for sensor placement optimization under uncertainty is outlined in this paper. The method consists of four components: (1) structural modeling and analysis, (2) probabilistic and reliability analyses, (3) damage detection, and (4) sensor placement optimization. The methodology is applied to the optimization of the sensor array of a SHM system for a TPS component. Specifically SPO under uncertainty was achieved. Further work is required in regards to extracting an optimum feature type as well as finding an optimum number of sensors. In addition, future work needs to incorporate sensor reliability and redundancy in the optimization.

5 ACKNOWLEDGEMENT

This research is sponsored by the United States Air Force Research Laboratory (project monitor: Mark Derriso) through subcontract to Anteon Corporation. The authors gratefully acknowledge this support.

REFERENCES

[1] Shkarayev, S., Krashantisa, R., and A. Tessler, "An Inverse Interpolation Method Utilizing In-Flight Strain Measurements for Determining Loads and Structural Response of Aerospace Vehicles." 3rd IWSHM, Stanford, CA, September 12-14, 2001.
[2] MSC.Software, *MSC.Nastran*. MSC.Software Corporation, Santa Ana, CA, 2001.
[3] MSC.Software, *MSC.Dytran*. MSC.Software Corporation, Santa Ana, CA, 2002.
[4] Ansys. ANSYS Release 9.0 Documentation. ANSYS, Inc., 2004.
[5] Abaqus. ABAQUS/Standard Version 6.5. ABAQUS, Inc., 2006.
[6] Dhondt, G., and K. Wittig, *CalculiX Version 1.4*, http://www.calculix.de, 1998.
[7] Forssell, J. *Imapct*. GNU General Public License, 2002.
[8] Edwins, D.J., *Modal Testing: Theory, Practice and Application*. 2nd Ed., Research Studies Press LTD., Philadephia, PA, 2000.
[9] Rebba, R., and S. Mahadevan, "Probabilistic Assessment of CAE Models." Proceedings of SAE 2006 World Congress, paper no. 2006-01-0226, Detroit, MI, April 3-6, 2006.
[10] Haldar, A., and S. Mahadevan, *Reliability Assessment Using Stochastic Finite Element Analysis.* John Wiley & Sons, Inc., New York, 2000.
[11] Banks, H.T., Joyner, M.L., Bincheski, B., and W.P. Winfree, "Real time computational algorithms for eddy-current-based damage detection." *Institute of Physics Publishing*, Vol. 18, pp. 795-823, 2002.
[12] Lew, J.S., and J.N. Juang, "Structural Damage Detection Using Virtual Passive Controllers." NASA/TM-2001-211251, December 2001.
[13] Doebling, S.W., Farrar, C.R., Prime, M.B., and D.W. Shevitz, *Damage Identification and Health Monitoring of Structures and Mechanical Systems from Changes in Their Vibration Characteristics: A Literature Review.* Los Alamos National Laboratory, Los Alamos, 1996.
[14] DeSimio, M., Miller, I., Derriso, M., Brown, K., and M. Baker, "Structural Health Monitoring Experiments with a Canonical Element of an Aerospace Vehicle." in proceedings 2003 IEEE Aerospace Conference, Big Sky, Montana, March 8-15, 2003.
[15] Padula, L.S., and R.K. Kincaid, "Optimization Strategies for Sensor and Actuator Placement." NASA/TM-1999-209126, April 1999.
[16] Padula, L.S., and R.K. Kincaid, "Optimal Sensor/Actuator Locations for Active Structural Acoustic Control." 39th AIAA, Long Beach, CA, April 20-23, 1998.
[17] Padula, L.S., and R.K. Kincaid, "Aerospace Applications of Integer and Combinatorial Optimization." NASA TM-110210, October 1995.
[18] Raich, A.M., and T.R. Liszkai, "Multi-Objective Genetic Algorithm Methodology for Optimizing Sensor Layouts to Enhance Structural Damage Detection." 4th IWSHM, Stanford, CA, September 15-17, 2003.
[19] Huyer W., and A. Neumaier, "SNOBFIT – Stable Noisy Optimization by Branch and Fit." submitted for publication in ACM Transactions on Mathematical Software, Vol. V, No. N, Month 20YY, http://www.mat.univie.ac.at/~neum/software/snobfit/.
[20] Olson, S., DeSimio, M., and M. Derriso, "Fastener Damage Estimation in a Square Aluminum Plate." submitted to *Structural Health Monitoring Journal*, 13 May, 2005.
[21] Guratzsch, R.F., Mahadevan, S., Olson, S., DeSimio, M., and M. Derriso. "Metrics for Validation of Modal Analysis Under Uncertainty." submitted for review to *Journal of Vibration and Control,* February 20, 2006.
[22] Shinozuka, M., and Deodatis, G., 1991, "Simulation of stochastic processes by spectral representation" Appl. Mech. Rev., 44, 4, 191-204.
[23] VanMarcke, E., Random Fields: Analysis and Synthesis, The MIT Press, Cambridge, MA, 1983.
[24] The Royal Aeronautical Society, Material Properties Handbook: Volume I Aluminum Alloys, North Atlantic Treaty Organization Advisory Group for Aerospace Research and Development, London, with Amendments from 1959 and 1966.
[25] Therrien, C.W., *Decision Estimation and Classification: An Introduction to Pattern Recognition and Related Topics*. John Wiley & Sons, New York, NY, 1989.
[26] Duda, R.O., Hart, P.E., and D.G. Stork, *Pattern Classification*. John Wiley & Sons, Inc., New York, 2001.